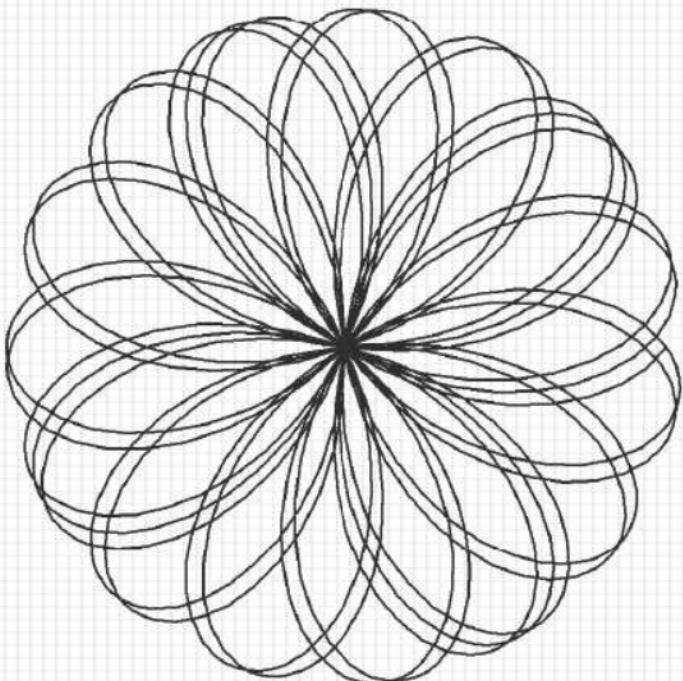


## 10.6: The Calculus of Polar Curves – Arc Length



$$r = 2 \sin(2.15\theta)$$
$$0 \leq \theta \leq 16\pi$$

To find the length of a curve in parametric mode:

$$\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

Polar is parametric with parameter  $\theta$ :

$$\int_{\theta_1}^{\theta_2} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} d\theta$$

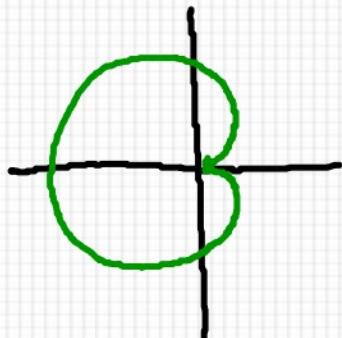
If we do a bunch of simplifying (proof omitted), we get:

$$\boxed{\int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta}$$



Ex: (Non calc) Find the length of the polar curve  $r = 1 - \cos\theta$ .

$$\frac{dr}{d\theta} = \sin\theta$$



REDUCTION  
IDENT. VARIATION

$$\int_0^{2\pi} \sqrt{(1-\cos\theta)^2 + \sin^2\theta} d\theta$$

$$\int_0^{2\pi} \sqrt{1-2\cos\theta + \cos^2\theta + \sin^2\theta} d\theta$$

$$\int_0^{2\pi} \sqrt{2-2\cos\theta} d\theta$$

$$\int_0^{2\pi} \sqrt{4\sin^2\frac{\theta}{2}} d\theta$$

$$2 \int_0^{\pi} 2\sin\frac{\theta}{2} d\theta$$

$$-4[2\cos\frac{\theta}{2}]_0^\pi$$

$$-8[\cos\frac{\theta}{2}]_0^\pi$$

$$-8(0-1)$$

$$\boxed{8}$$



Ex: (Non calc) Find the length of the polar curve  $r = e^\theta$  from  $\theta = 0$  to  $\theta = 1$ .

$$\int_0^1 \sqrt{(e^\theta)^2 + (e^\theta)^2} d\theta$$

$$\int_0^1 \sqrt{2e^{2\theta}} d\theta$$

$$\sqrt{2} \int_0^1 e^\theta d\theta$$

$$\left[ \sqrt{2} e^\theta \right]_0^1$$
$$\boxed{\sqrt{2}(e - 1)}$$



# **Homework:**

Section 10.6 WS – Arc Length in Polar  
Chapter 10 AP Packet (Polar) - #16

